liquids involving parameters for intermolecular potential energy and for molecular rotation. When applied to binary diffusion, this procedure entails rational mixing rules for the parameters associated with both components. It is obvious that the pseudocritical method is advantageous owing to its simplicity as well as generality.

It is noted that pressure does not play a role simply because slight pressures (less than 5×10^5 N/m^2) were applied only when required to prevent the liquid from vaporizing as the temperature was raised above its normal boiling point; diffusivity in liquid is known to be a weak function of pressure.

Further studies are under way which will help to establish the conditions for the applicability of the proposed correlation. For example, restrictions on the shape and polarity of both solute and solvent molecules can be removed and the range of molecular weights extended to test the validity of Eq. 1.

NOTATION

D = tracer diffusivity of gases in liquids, m^2/s
T = absolute temperature, K

Subscripts

c = pseudocritical conditions

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Local Heat Transfer Coefficients for a Horizontal Tube in a Large-Particle Fluidized Bed at Elevated Temperature

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Current interest in large-particle fluidized beds stems from the development of several new processes. The fluidized combustion of coal, in which crushed coal is burned in a bed of dolomite or limestone particles, is possibly the most important. Recent studies (Leon et al., 1980; Tang and Howe, 1980; Goblirsch et al., 1980; Skinner, 1971) all report promising results based on pilot-plant studies. Heating of process fluids (e.g., crude oil) is being studied by Cherrington et al. (1977).

In all of the above-mentioned applications, horizontal tube arrays have proven to be a practical configuration for the immersed heat exchanger surface. A review of literature on the topic of heat transfer to tubes immersed in fluidized beds is given by Saxena et al. (1978).

The present publication reports local heat transfer coefficients for a single 50.8 mm diameter horizontal tube in a large-particle (d_p = 3.23 mm maximum), high-temperature (T_B = 1,053 K maximum) fluidized bed. Both the thermal stress problem for the tube wall and the analysis of coking of process fluids within the tube require local bed-to-tube heat transfer information.

Details concerning the instrumentation used as well as data acquisition and data reduction techniques appear elsewhere (George, 1981). Some useful information about the fluidized-bed facility used in the study are given below.

FLUIDIZED BED

Cross-sectional dimensions of the fluidized bed were 0.60 m × 0.30 m. Combustion products of propane were used to heat the bed. A proportional-type controller regulated the propane flow rate to maintain the desired combustion chamber exit temperature. No combustion occurred in the bed itself. A complete description of the fluidized-bed facility, including engineering drawings, is given by Welty (1978). The design of the distributor plate is given by George (1981).

Granular refractory material, with commercial designation Ione Grain A, was used for bed material. Chemical composition of this material was given by the supplier as: 53.5% silica, 43.8% alumina, 2.3% titania, and 0.4% other. The particles had a smooth surface and fairly spherical shape from several previous test runs. Particle density was 2,700 kg/m^3.

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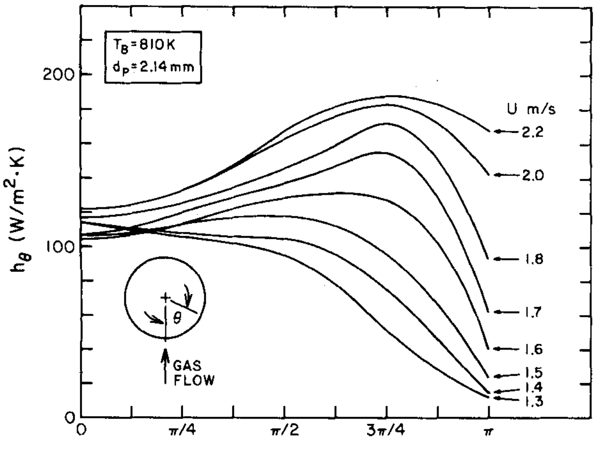

Figure 1. Local heat transfer coefficient vs. angular position for d_p = 2.14 mm and T_B = 810 K.

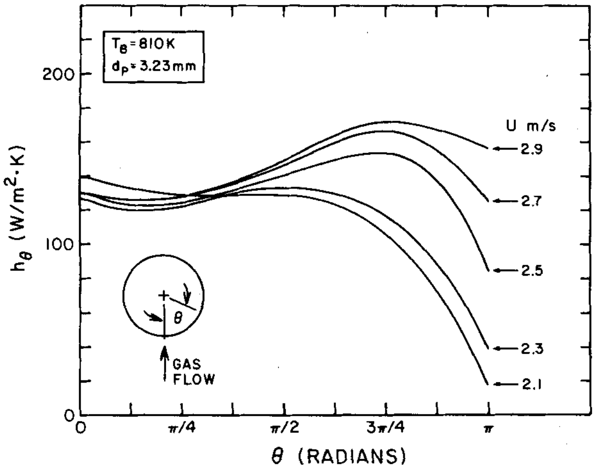

Figure 2. Local heat transfer coefficient vs. angular position for d_p = 3.23 mm and T_B = 810 K.

No reliable data for the thermal conductivity or specific heat of Ione Grain A are known to the writers; however, Chung et al. (1972) and Ziegler et al. (1964) found the thermal conductivity of the particle to be an insignificant factor concerning heat transfer to surfaces immersed in fluidized beds. For large-particle beds, an insignificant dependence on thermal conductivity and volumetric heat capacity of the particles was found by Zabrodsky et al. (1978). The surface mean diameter, d_p, was computed as in Kunii and Levenspiel (1969). A screen anlaysis of the particles appears elsewhere (George, 1981).

Packed bed height was 0.50 m for all cases reported. The centerline of the instrumented tube was located 0.31 m above the distributor plate.

EXPERIMENTAL RESULTS

Fluidized beds of large particles with wide-size spectrum, as used here, have no well-defined minimum fluidization velocity (Botterill and Teoman, 1980; Doicher and Akhmakov, 1979, Babu et al., 1978; Wright et al., 1970). Partially for this reason, the data are presented in dimensional form rather than using a nondimensional velocity coordinate U/U_{mf}.

Figures 1, 2, 3 and 4 show the local heat transfer coefficient for both particle sizes and bed temperatures considered in this study. The bed temperature was controlled to within ±6 K of the specified value for all cases reported.

The tube wall temperature, averaged around the outside periphery of the tube, varied from 425 to 480 K with T_B = 1,052 or

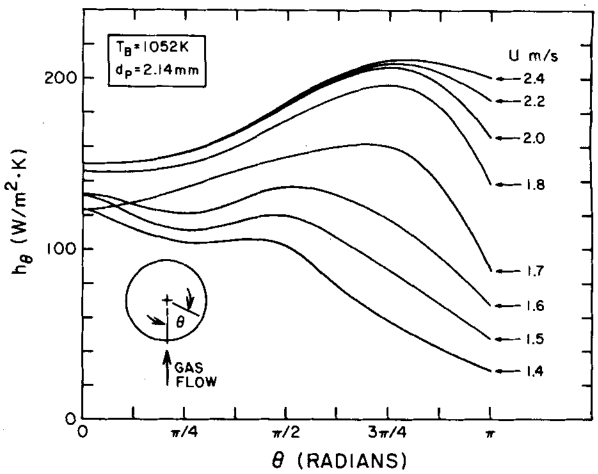

Figure 3. Local heat transfer coefficient vs. angular position for d_p = 2.14 mm and T_B = 1,052 K.

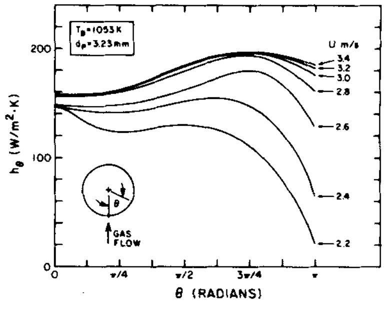

Figure 4. Local heat transfer coefficient vs. angular position for d_p = 3.23 mm and T_B = 1,053 K.

1,053 K, and from 385 to 420 K with T_B = 810 K. More precise values of the tube wall temperature, as a function of fluidizing conditions, are given by George (1981).

Probably the most obvious trend present in the local heat transfer coefficient data is the very large (factor of eight or more) change in the magnitude of the coefficient at the $\theta = \pi$ radian position, as the superficial gas velocity is varied. This is probably due to a relatively cool defluidized stack of particles, which remains on the top of the tube at low gas velocities. Intense bubbling, at higher gas velocities, displaces this defluidized stack of particles, and the local heat transfer coefficient increases greatly. This explanation is consistent with the hydrodynamic information (local voidage, emulsion residence time, and visual inspection) available in the literature (Syromyatnikov et al., 1977; Glass and Harrison, 1964; Loew et al., 1979; Hager and Schrag, 1976; Catipovic, 1979).

COMPARISON OF PRESENT RESULTS WITH OTHER DATA

Little heat transfer information is available in the literature for the geometry, particle size range, and bed temperatures considered here. To the authors' knowledge, no local heat transfer coefficient data for the conditions considered here, have been reported. Thus, the spatial average heat transfer coefficient must be used for purposes of comparison. The spatial average heat transfer coefficient was computed by applying the trapezoidal rule for numerical quadrature (e.g., Hamming, 1973) to the local values of the heat transfer coefficient.

Grewal and Saxena (1981) have written a comprehensive paper on the topic of maximum heat transfer coefficients for single horizontal tubes immersed in fluidized beds. They emphasized small-particle beds (d_p < 1 mm), but present some data for large-particle beds (obtained from other literature) as well. For all data considered, they found good correlation between Nu_{MAX} and Ar. While Grewal and Saxena recommend their own more complex

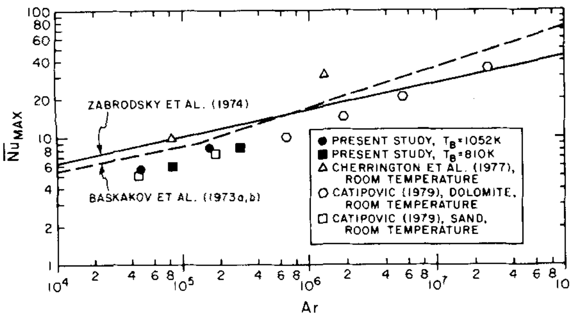

Figure 5. Maximum spatial-average Nusselt number vs. Archimedes number data for several studies.

correlation, the earlier correlation due to Zabrodsky et al. (1974) and stated below fits the available data nearly as well

$$\overline{Nu}_{MAX} = 0.88\ Ar^{0.213}$$

Extrapolation of this correlation to values of Ar greater than 10^5 is not recommended; that is the highest value of Ar considered by Grewal and Saxena.

Other correlations, valid for the Archimedes number range considered here, were developed by Baskakov et al. (1973a,b).

Almost all of the data correlated by Zabrodsky et al. (1974) and Baskakov et al. (1973a,b) were obtained at near room temperature with air as a fluidizing gas. These correlations, along with other high Archimedes number data, are compared with results of the present study in Figure 5. All gas (air) properties have been evaluated at the bed temperature. Property data were obtained from Touloukian et al. (1970, 1975).

The data reported here are in acceptable agreement with those reported by other investigators.

ACKNOWLEDGMENT

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NOTATION

Ar $= \dfrac{g d_p^3(\rho_s - \rho_f)}{\rho_f \nu_f^2}$ Archimedes number
d_p = surface mean particle diameter
g = acceleration due to gravity
$\bar{h}$ = spatial-average bed-to-tube heat transfer coefficient
$\bar{h}_{MAX}$ = maximum spatial-average bed-to-tube heat transfer coefficient
h_θ = local bed-to-tube heat transfer coefficient
k_f = thermal conductivity of fluidizing gas at the bed temperature
$\overline{Nu}_{MAX}$ $= \dfrac{d_p \bar{h}_{MAX}}{k_f}$ Maximum spatial-average Nusselt Number
T_B = temperature of fluidized bed
U = superficial gas velocity
U_{mf} = superficial gas velocity at minimum fluidization conditions

Greek Letters

θ = angular position on tube surface
ν_f = kinematic viscosity of fluidizing gas at the bed temperature
ρ_s = density of particle
ρ_f = density of fluidizing gas at the bed temperature

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Effect of Elasticity on Mixing Torque Requirements for Rushton Turbine Impellers

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The calculation of torque and power required to mix viscoelastic fluids is an important problem in the design of bulk polymerization reactors, equipment for compounding polymeric fluids, vessels for blending polymer-stabilized latex paints, and fermenters for polysaccharide production. In this note we present a correlation that allows one to assess the effect of fluid elasticity on the torque (or power) required for mixing viscoelastic fluids.

Most work on non-Newtonian mixing is based on papers by Metzner and Otto (1961, 1957) which consider the mixing of a purely-viscous, pseudo-plastic fluid. Their analysis involves obtaining a torque vs. Reynolds number correlation for a vessel from data taken with a Newtonian fluid. An "effective shear rate" in the vessel is determined, and from the viscosity vs. shear rate curve for the non-Newtonian fluid of interest the "effective viscosity" at that "effective shear rate" is determined. A Reynolds number is then calculated using this viscosity and shear rate and this Reynolds number is used in the *Newtonian fluid correlation* to determine the torque. The effects of elasticity are not considered in this analysis, but since elasticity is known to produce differences in flow fields around mixing impellers (White et al., 1977) it is important to known under what conditions torque correlations for viscoelastic fluids differ significantly from those of Newtonian fluids.

An overview of earlier work on the effect of elasticity on mixing non-Newtonian fluids has been presented in the review by Ulbrecht (1974). Previous attempts to draw conclusions about the effect of elasticity on torque have been confounded because most viscoelastic fluids have a strongly shear-thinning viscosity as well as elasticity and, therefore, changes in torque may be due to changes in fluid viscosity or elasticity. It has not been possible to uncouple those two effects. Recently, however, Boger (1977/78) and Walker, Gorell and Homsy (1980) have presented data on a model fluid having a constant viscosity, but having elasticity (primary normal stress difference) that can be varied. This is the fluid we have used. Figure 1 shows the viscosity and primary normal stress difference for three fluids with three levels of polyacrylamide (Dow Chem. Co. Separan AP30) in a solution of corn syrup (CPC International corn syrup #1132) glycerine, and water. A 50/50 solution of glycerine and water was added to the corn syrup to adjust the viscosity to 4.0 Pa • s. Figure 1 shows that the viscosities of the three fluids are constant over the range of shear rate of interest. Without the addition of polyacrylamide the fluid has no measurable primary normal stress. With the addition of 0.015 wt. % polyacrylamide the fluid has significant normal stress and doubling the polyacrylamide concentration (0.03 wt. %) increases the normal stress by an order of magnitude. Using these fluids we can assess the importance of fluid elasticity on mixing torque requirements.

The apparatus used, Figure 2, consists of a 1/4 hp (0.19 kW) variable speed motor, an inductive pick-up tachometer sensing the motion of a 60-tooth sprocket gear, a 0.225 m diameter by 0.263 m tall fully-baffled glass tank, and an air-bearing table with a bonded strain gauge transducer to measure torque. The air-bearing table ensures that there is negligible frictional drag so that low torques can be measured accurately from 0 to 1.2 N•m with an error of less than 0.1% of full scale. Motor speed can be varied from about 20 to 170 rpm. The six-bladed Rushton turbine impeller studied had a 7.5 cm diameter disk with 2.5 cm long by 1.8 cm wide blades attached around the edge.

Figure 3 shows the dimensionless torque versus Reynolds number

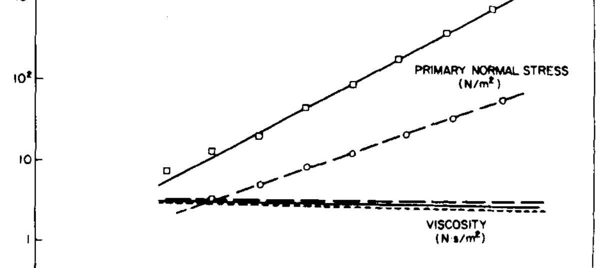

Figure 1. Viscosity and primary normal stress difference for three constant-viscosity, elastic fluids. The fluid with 0% polyacrylamide (△) has no normal stress, the fluid with 0.015% wt. polyacrylamide (○) has the same constant viscosity but significant normal stress, the fluid with 0.03% wt. polyacrylamide (□) has an order of magnitude greater normal stress.

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